H infinity observer for time-delay systems. Application to FDI for irrigation canals

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Abstract— **This paper deals with the problem of fault detection and isolation for time-varying delayed systems. It consists to develop a H_∞ observer that generates residuals sensitive to some faults and insensitive to others in order to detect and isolate actuator faults which can occur on the regulation gates of an irrigation canal. The observer design uses a simplified approximate model of the Saint-Venant equations and is formulated with delay-dependent Linear Matrix Inequality (LMI). Simulations done with a realistic model of a real canal show the effectiveness of the method.**

I. INTRODUCTION

Irrigation represents more than 80% of world fresh water consumption, and large water losses occur in irrigation canals due to poor management. Water efficiency can be improved by the integration of automatic control in the management of such systems. This automation requires a supervision to inspect the presence of faults. The motivation of this paper is then to achieve a fault detection and isolation for an irrigation canal, which is a time-delayed dynamical system.

In the automatic field, a lot of research was focused on fault detection and isolation (FDI) for Linear Time-Invariant systems (LTI) (see e.g. [1] for a comprehensive review) but few studies considered time-delay systems.

In [9], a diagnosis scheme based on a bank of an Unknown Inputs Observers (UIO) was achieved for systems with delayed state and input to realize the FDI of unknown gates faults for an irrigation canal. This work required a perfect decoupled faults condition. However, this condition is not always satisfied in practice. In this case, an alternative is to use a H_∞ technique. The original system is decomposed into several sub-systems, each being sensitive to a sub-set of faults defined beforehand, whilst minimising the other faults. Similar techniques have been developed for LTI systems, see e.g. [5], [6].

The main goal of this paper is to develop a H_∞ observer for systems with time-varying delays in both state and inputs, and then to use it for FDI problems related to irrigation canals. The system considered here consists of two pools in series, leading to a system with two delays, but the results can be generalized to a larger number of delays. The observer design relies on the Integrator-Delay (ID) model which is a well approximate model of Saint-Venant equations. The problem is formulated using the delay-dependent conditions given in [10] which are less conservative than previous work [4], [8], which are extended to the case of two delays and H_∞ problem. It is expressed in terms of Linear Matrix Inequalities (LMI) and is computed numerically using .

The paper is organized as follows: section II presents the problem formulation and recalls the results already obtained for perfect decoupling. Section III gives the synthesis of the H_∞ observer. Section IV details the FDI scheme and section V shows simulations obtained from the application of such FDI on a realistic model of an irrigation canal.

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II. PROBLEM STATEMENT

A. Channel hydraulic model

An irrigation canal can be represented as a series of pools separated by regulation gates. Each pool is represented by the integrator-delay model [7]:

$$y_i(s) \;=\; \tfrac{a_i}{s} e^{-\tau_i s} q_i(s) - \tfrac{a_i}{s}(q_{i+1}(s) + p_{i+1}(s)) \qquad (1)$$

where y_i is the downstream water level of the pool i, q_i and q_{i+1} are the upstream and downstream water flow rate deviations from the equilibrium flow rates, respectively, a_i is the inverse of the backwater area of pool i, and p_{i+1} is the unmeasured water withdrawal, occurring at the downstream end of pool i.

Each regulation gate is represented by its linearized model around the equilibrium:

$$q_{i+1}(s) = b_i y_i(s) + k_{i+1} u_{i+1}(s) + \bar{b}_i \bar{y}_i(s) \qquad (2)$$

where u_i is the control input and $\bar{y}_i$ is the water level deviation at the downstream position of the regulation gate i.

B. Modelling of possible faults

We consider that the canal is managed with the classical distant downstream control framework, where the actuators are the gate openings, and the sensors are the upstream and downstream water levels at each gate. In this context, we consider three different kinds of faults that can affect the controlled system:

- actuator fault (e.g. a floating object that tampers the gate), which is modelled by a bias on the inputs (denoted δu),
- sensor fault (here on the water level measurements), which is modelled by a bias on the output (denoted δy),
- unmeasured discharge withdrawal occurring at the downstream end of the pool i (denoted p_{i+1}).

These three different kind of faults can be considered in the same framework. Indeed, based on eq. (2), we can show that a fault affecting the gate opening u has the same effect on the discharge q as a fault affecting the measured downstream level $\bar{y}$, the measured upstream level y or the discharge q (withdrawal p).

Without any loss of generality, we therefore consider faults occurring on the actuators, keeping in mind the fact that the other faults can be considered equivalently using exactly the same structure.

C. Dimensionless model

1) Dimensionless irrigation canal model: In order to get generic results, we express the system in terms of dimensionless data, denoted with a superscript *. To this end, let τ_m denote the largest time delay. Then, defining:

$$s^* = \tau_m s, y_{r_i} = u_{r_i} = a_i \tau_m Q_0, y_i^* = \frac{y_i}{y_{r_i}}, \bar{y}_i^* = \frac{\bar{y}_i}{y_{r_i}}, u_i^* = \frac{u_i}{u_{r_i}}$$

and $q_i^* = \frac{q_i}{Q_0}$, the equations (1) and (2) become:

$$y_i^*(s^*) \;=\; \tfrac{1}{s^*} e^{-\tau_i^* s^*} q_i^*(s^*) - \tfrac{1}{s^*} q_{i+1}^*(s^*) \qquad (3)$$

and

$$q_{i+1}^*(s^*) = b_i^* y_i^*(s^*) + k_{i+1}^* u_{i+1}^*(s^*) + \bar{b}_i^* \bar{y}_i^*(s^*) \qquad (4)$$

Where

$$\tau_i^* = \frac{\tau_i}{\tau_m}, b_i^* = b_i \frac{y_r}{Q_0}, \bar{b}_i^* = \bar{b}_i \frac{y_r}{Q_0},$$

and $k_i^* = k_i \frac{y_r}{Q_0}$. Combining (3) and (4), we obtain the dynamic relationship for each pool between inputs and outputs:

$$\begin{array}{l} y_i^*(s^*) = \frac{1}{s^*} e^{-\tau_i^* s^*} (b_{i-1}^* y_{i-1}^* + k_i^* u_i^* + \bar{b}_{i-1}^* \bar{y}_{i-1}^*) \\ \qquad - \frac{1}{s^*} (b_i^* y_i^* + k_{i+1}^* u_{i+1}^* + \bar{b}_i^* \bar{y}_i^*) \end{array} \qquad (5)$$

where $t^* = \frac{t}{\tau_m}$.

In the time domain, this leads to the following delay-differential equation:

$$\begin{array}{rl} \frac{dy_i^*}{dt^*}(t^*) = & b_{i-1}^* y_{i-1}^*(t^* - \tau_i^*) + k_i^* u_i^*(t^* - \tau_i^*) - \bar{b}_i^* \bar{y}_i^*(t^*) \\ & + \bar{b}_{i-1}^* \bar{y}_{i-1}^*(t^* - \tau_i^*) - b_i^* y_i^*(t^*) - k_{i+1}^* u_{i+1}^*(t^*) \end{array} \qquad (6)$$

For the sake of simplicity and without loss of generality, we consider a canal with two pools, with three regulation gates acting on the boundaries of each pool (see figure 1).

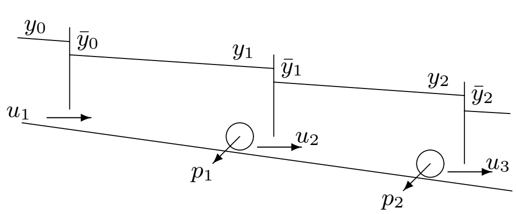

Fig. 1. Schematic view of a canal with two pools

In this case, we obtain the two following equations:

$$\begin{array}{l} \frac{dy_1^*(t^*)}{dt^*} = b_0^* y_0^*(t^* - \tau_1^*) + k_1^* u_1^*(t^* - \tau_1^*) + \bar{b}_0^* \bar{y}_0^*(t^* - \tau_1^*) \\ \qquad - b_1^* y_1^*(t^*) - k_2^* u_2^*(t^*) - \bar{b}_1^* \bar{y}_1^*(t^*) \end{array} \qquad (7)$$

$$\begin{array}{l} \frac{dy_2^*(t^*)}{dt^*} = b_1^* y_1^*(t^* - \tau_2^*) + k_2^* u_2^*(t^* - \tau_2^*) + \bar{b}_1^* \bar{y}_1^*(t^* - \tau_2^*) \\ \qquad - b_2^* y_2^*(t^*) - k_3^* u_3^*(t^*) - \bar{b}_2^* \bar{y}_2^*(t^*) \end{array} \qquad (8)$$

We assume that $y_0^* = 0$, $\bar{y}_2^* = 0$ and $u_3^* = 0$, which corresponds to the realistic case of a canal managed with distant downstream control [3]. The last equations are then reduced to:

$$\begin{array}{l} \frac{dy_1^*(t^*)}{dt^*} = k_1^* u_1^*(t^* - \tau_1^*) + \bar{b}_0^* \bar{y}_0^*(t^* - \tau_1^*) \\ \qquad - b_1^* y_1^*(t^*) - k_2^* u_2^*(t^*) - \bar{b}_1^* \bar{y}_1^*(t^*) \end{array} \qquad (9)$$

$$\begin{array}{l} \frac{dy_2^*(t^*)}{dt^*} = b_1^* y_1^*(t^* - \tau_2^*) + k_2^* u_2^*(t^* - \tau_2^*) + \bar{b}_1^* \bar{y}_1^*(t^* - \tau_2^*) \\ \qquad - b_2^* y_2^*(t^*) \end{array} \qquad (10)$$

The corresponding state representation is a system with delayed state and inputs given by:

$$\begin{array}{rl} \dot{x}^*(t^*) = & \sum_{i=0}^{2} A_i^* x^*(t^* - \tau_i^*) + \sum_{i=0}^{2} B_i^* u^*(t^* - \tau_i^*) \\ & + \sum_{i=0}^{2} \bar{B}_i^* \bar{y}^*(t^* - \tau_i^*) \\ y^*(t^*) = & C^* x^*(t^*) \end{array} \qquad (11)$$

where $x^* = \begin{pmatrix} y_1^* & y_2^* \end{pmatrix}^T \in \Re^n$ is the state, $u^* = \begin{pmatrix} u_1^* & u_2^* \end{pmatrix}^T \in \Re^{n_u}$ is the control, $\bar{y}^* = \begin{pmatrix} \bar{y}_0^* & \bar{y}_1^* \end{pmatrix}^T \in \Re^m$ are the measured inputs. A_i^*, B_i^*, $i = 0, 1$ and C are known real constant matrices defined by:

$$A_0^* = \begin{pmatrix} -b_1^* & 0 \\ 0 & -b_2^* \end{pmatrix}, A_1^* = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, A_2^* = \begin{pmatrix} 0 & 0 \\ b_1^* & 0 \end{pmatrix}$$

$$B_0^* = \begin{pmatrix} 0 & -k_2^* \\ 0 & 0 \end{pmatrix}, B_1^* = \begin{pmatrix} k_1^* & 0 \\ 0 & 0 \end{pmatrix}, B_2^* = \begin{pmatrix} 0 & 0 \\ 0 & k_2^* \end{pmatrix}$$

$$\bar{B}_0^* = \begin{pmatrix} 0 & -\bar{b}_1^* \\ 0 & 0 \end{pmatrix}, \bar{B}_1^* = \begin{pmatrix} \bar{b}_0^* & 0 \\ 0 & 0 \end{pmatrix}, \bar{B}_2^* = \begin{pmatrix} 0 & 0 \\ 0 & \bar{b}_1^* \end{pmatrix}$$

and $C^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

2) Dimensionless model of the faults: The faults affecting the actuators are then represented by $u_f^*(t) = u^*(t) + f^*(t)$. The system representation including faults reads:

$$\begin{array}{rl} \dot{x}^*(t^*) = & \sum_{i=0}^{2} A_i^* x^*(t^* - \tau_i^*) + \sum_{i=0}^{2} B_i^* u^*(t^* - \tau_i^*) \\ & + \sum_{i=0}^{2} \bar{B}_i^* \bar{y}^*(t^* - \tau_i^*) + E^* f^*(t^*) \\ y^*(t^*) = & C x^*(t^*) \end{array} \qquad (12)$$

Where E^* and f^* are defined in [9].

3) Considered model: We can now put this problem into a more general form of system with delayed states and inputs with the presence of faults, to develop methods for FDI for delayed systems:

$$\begin{array}{rl} \dot{x}(t) = & \sum_{i=0}^{2} A_i x(t - \tau_i(t)) + \sum_{i=0}^{2} B_i u(t - \tau_i(t)) + E f(t) \\ y(t) = & C x(t) \end{array} \qquad (13)$$

with

$$0 \le \tau_i(t) \le h_i, 0 \le \dot{\tau}_i(t) \le d_i$$

This enables us to consider a more general case where delays are bounded, but possibly time-varying. Moreover, only two delays are considered here because of our system'structure but the designed method can be easily extended to the case of multiple delays.

4) UIO for time-delay system: From [9], a UIO exists if and only if the decoupled unknown input condition (14) holds

$$rank(CE) = rank(E) \tag{14}$$

Unfortunately, this condition does not always hold in practice. Consequently, we are interested in developing an observer which does not need to satisfy this condition. So, in the following, we will develop an H_∞ observer of the following form:

$$\begin{cases} \dot{z}(t) = & \sum_{i=0}^{2} F_i z(t-\tau_i(t)) + \sum_{i=0}^{2} TB_i u(t-\tau_i(t)) \\ & + \sum_{i=0}^{2} G_i y(t-\tau_i(t)) \\ \hat{x}(t) = & z(t) + Ny(t) \\ \hat{y}(t) = & C\hat{x}(t) \\ r(t) = & y(t) - \hat{y}(t) \end{cases} \tag{15}$$

Where z is the state of the observer, $\hat{x}$ is the estimated state of the system, r is the output of the observer, $\hat{y}$ is the estimated output, u and y are the inputs of the observer.

III. H_∞ OBSERVER DESIGN

A. Observer design

The H_∞ observer problem for a performance level $\gamma > 0$ is to find the gain of the observer (15) that stabilizes the state estimation error and satisfies the inequality

$$J = \int_0^\infty r^T(t)r(t) - \gamma^2 f^T(t)f(t)dt < 0 \tag{16}$$

Defining $e(t)$ as the error between $x(t)$ and its estimation $\hat{x}(t)$, its derivative is given by:

$$\begin{array}{l} \dot{e}(t) = \sum_{i=0}^{2} F_i e(t-\tau_i(t)) + TEf(t) \\ r(t) = Ce(t) \end{array} \tag{17}$$

Then, we have the following corollary (see [9] for the proof).

Corollary 1: The observer (15) stabilizes the state estimation error and ensures the performance index (16) if :

1) $\dot{e}(t) = \sum_{i=0}^{2} F_i e(t-\tau_i(t)) + TEf(t)$ is stable
2) $T + NC = I_n$
3) $\bar{G}_i = G_i - F_i N, \quad i = 0,1,2$
4) $F_i = TA_i - \bar{G}_i C, \quad i = 0,1,2$
5) $\|T_{fr}(s)\|_\infty < \gamma$

From corollary 1, the design of the observer (15) is reduced to find the matrices $T, N, F_i, \bar{G}_i, G_i$ so that conditions 1)-4) are satisfied. As in [9], we write the algebraic constraints 2) and 4) in a matrix form:

$$\begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{bmatrix} \Theta_1 = \Psi_1 \tag{18}$$

where

$$\Theta_1 = \begin{bmatrix} I_n & A_0 & A_1 & A_2 \\ C & 0 & 0 & 0 \\ 0 & -I_n & 0 & 0 \\ 0 & -C & 0 & 0 \\ 0 & 0 & -I_n & 0 \\ 0 & 0 & -C & 0 \\ 0 & 0 & 0 & -I_n \\ 0 & 0 & 0 & -C \end{bmatrix} \in \Re^{(4n+4p)\times(4n)}$$

and

$$\Psi_1 = \begin{bmatrix} I_n & 0 & 0 & 0 \end{bmatrix} \in \Re^{n\times(4n)}.$$

If the condition $rank\begin{bmatrix} \Theta_1 \\ \Psi_1 \end{bmatrix} = rank\Theta_1$ holds, then (18) admits the general solution

$$\begin{array}{c} \begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{bmatrix} = \\ \Psi_1\Theta_1^+ - K\left(I_{4(n+p)} - \Theta_1\Theta_1^+\right) \end{array} \tag{19}$$

where Θ_1^+ is the generalized inverse matrix of Θ_1 and K is a free matrix to be fixed in order to achieve the remaining conditions 1) and 5). Letting

$$\begin{array}{lcl} \varphi_0^T & = & \begin{bmatrix} A_0^T & 0 & 0 & C^T & 0 & 0 & 0 & 0 \end{bmatrix} \\ \varphi_1^T & = & \begin{bmatrix} A_1^T & 0 & 0 & 0 & 0 & C^T & 0 & 0 \end{bmatrix} \\ \varphi_2^T & = & \begin{bmatrix} A_2^T & 0 & 0 & 0 & 0 & 0 & 0 & C^T \end{bmatrix} \\ \varphi_T^T & = & \begin{bmatrix} I_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \end{array}$$

the condition (4) and the relation TE can be written as:

$$\begin{array}{c} \begin{bmatrix} T & F_0 & F_1 & F_2 \end{bmatrix} = \\ \begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{bmatrix} \times \\ \begin{bmatrix} \varphi_0 & \varphi_1 & \varphi_2 & \varphi_T \end{bmatrix} \end{array} \tag{20}$$

Inserting (19) into (20), we obtain

$F_0 = \chi_0 - K\beta_0, F_1 = \chi_1 - K\beta_1, F_2 = \chi_2 - K\beta_2$, and $T = \chi_T - K\beta_T$ where:

$$\begin{array}{ll} \chi_0 = \Psi_1\Theta_1^+\varphi_0, & \chi_1 = \Psi_1\Theta_1^+\varphi_1, \\ \chi_2 = \Psi_1\Theta_1^+\varphi_2, & \chi_T = \Psi_1\Theta_1^+\varphi_T, \\ \beta_0 = \left(I_{4(n+p)} - \Theta_1\Theta_1^+\right)\varphi_0, & \beta_1 = \left(I - \Theta_1\Theta_1^+\right)\varphi_1, \\ \beta_2 = \left(I - \Theta_1\Theta_1^+\right)\varphi_2, & \beta_T = \left(I - \Theta_1\Theta_1^+\right)\varphi_T \end{array}$$

Now, substituting each term by its expression in (17) and denoting $\chi_f = \chi_T E$ and $\beta_f = \beta_T E$, we obtain:

$$\begin{array}{l} \dot{e}(t) = \sum_{i=0}^{2}(\chi_i - K\beta_i)e(t-\tau_i(t)) + (\chi_f - K\beta_f)f(t) \\ r(t) = Ce(t) \end{array} \tag{21}$$

We state the following theorem:

Theorem 2: The error is asymptotically stable with the performance index $J < 0$ if for some scalars ϵ_i, $i = 1,6$, $\bar{\epsilon}_i$, $i = 1,2$ there exist matrices $Z_i > 0$, $S_i > 0$,$R_i > 0$, $Q_i > 0$, U_i, W_i, $i = 1,2$, H_i, $i = 1,7$, U, and $P > 0$ such that the following LMIs are satisfied:

$$\begin{bmatrix} Q_i & U_1 \\ U_i^T & R_i \end{bmatrix} \geq 0, i = 1,2 \tag{22}$$

$$\begin{bmatrix} \Phi & h_1\bar{H}_1 & h_2\bar{H}_2 \\ * & -h_1\bar{Z}_1 & 0 \\ * & * & -h_2\bar{Z}_2 \end{bmatrix} < 0 \tag{23}$$

Where, for $i = 1,2$:

$$\bar{Z}_i = \begin{bmatrix} S_i & W_1 \\ W_i^T & Z_i \end{bmatrix}, \bar{H}_i = \begin{bmatrix} -\bar{\epsilon}_i(P\chi_0 - U\beta_0)^T & H_1 \\ -\bar{\epsilon}_i(P\chi_1 - U\beta_1)^T & H_2 \\ -\bar{\epsilon}_i(P\chi_2 - U\beta_2)^T & H_3 \\ \bar{\epsilon}_i P & H_4 \\ 0 & H_5 \\ 0 & H_6 \\ -\bar{\epsilon}_i(P\chi_f - U\beta_f)^T & H_7 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix}$$

with

$\Phi_{11} = \sum_{i=1}^{2}(h_i S_i + Q_i) + \epsilon_1 Sym(P\chi_0 - U\beta_0) + 2Sym(H_1) + C^T C$

$\Phi_{12} = \epsilon_1(P\chi_1 - U\beta_1) + \epsilon_2(P\chi_0 - U\beta_0)^T - H_1 + 2H_2^T$

$\Phi_{13} = \epsilon_3(P\chi_0 - U\beta_0)^T + \epsilon_1(P\chi_2 - U\beta_2) + 2H_3^T - H_1$

$\Phi_{14} = P + \sum_{i=1}^{2}(U_i + h_i W_i) + 2H_4^T + \epsilon_4(P\chi_0 - U\beta_0)^T - \epsilon_1 P$

$\Phi_{15} = 2H_5^T + \epsilon_5(P\chi_0 - U\beta_0)^T$

$\Phi_{16} = 2H_6^T + \epsilon_6(P\chi_0 - U\beta_0)^T$

$\Phi_{17} = 2H_7^T + \epsilon_1(P\chi_f - U\beta_f)$

$\Phi_{22} = -(1-d_1)Q_1 - Sym(H_2) + \epsilon_2 Sym(P\chi_1 - U\beta_1)$

$\Phi_{23} = -H_3^T - H_2 + \epsilon_2(P\chi_2 - U\beta_2) + \epsilon_3(P\chi_1 - U\beta_1)^T$

$\Phi_{24} = -H_4^T + \epsilon_4(P\chi_1 - U\beta_1)^T - \epsilon_2 P$

$\Phi_{25} = -(1-d_1)U_1 - H_5^T + \epsilon_5(P\chi_1 - U\beta_1)^T$

$\Phi_{26} = -H_6^T + \epsilon_6(P\chi_1 - U\beta_1)^T$

$\Phi_{27} = -H_7^T + \epsilon_2(P\chi_f - U\beta_f)$

$\Phi_{33} = -(1-d_2)Q_2 - Sym(H_3) + \epsilon_3 Sym(P\chi_2 - U\beta_2)^T$

$\Phi_{34} = -H_4^T + \epsilon_4(P\chi_2 - U\beta_2)^T$

$\Phi_{35} = -H_5^T + \epsilon_5(P\chi_2 - U\beta_2)^T$

$\Phi_{36} = -(1-d_2)U_2 - H_6^T + \epsilon_6(P\chi_2 - U\beta_2)^T$

$\Phi_{37} = -H_7^T + \epsilon_3(P\chi_f - U\beta_f)$

$\Phi_{44} = \sum_{i=1}^{2}(R_i + h_i Z_i) + \epsilon_4 P$

$\Phi_{45} = \epsilon_5 P$

$\Phi_{46} = \epsilon_6 P$

$\Phi_{47} = \epsilon_4(P\chi_f - U\beta_f)$

$\Phi_{55} = -(1-d_1)R_1$

$\Phi_{56} = 0$

$\Phi_{57} = \epsilon_5(P\chi_f - U\beta_f)$

$\Phi_{66} = -(1-d_2)R_2$

$\Phi_{67} = \epsilon_6(\chi_f - U\beta_f)$

$\Phi_{77} = -\gamma^2 I_f$

The gain K is given by: $K = P^{-1}U$.

Proof: Consider the Lyapunov-Krasovskii function :

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

with $V_1(t) = e(t)^T P e(t)$,

$$V_2(t) = \sum_{i=1}^{2}\int_0^{h_i}\int_{t-\theta}^{t} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix}^T \begin{bmatrix} S_i & W_i \\ W_i^T & Z_i \end{bmatrix} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix} ds d\theta,$$

$$V_3(t) = \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix}^T \begin{bmatrix} Q_i & U_i \\ U_i^T & R_i \end{bmatrix} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix} ds$$

We have:

$$\begin{aligned} &\dot V(t) = 2e^T P\dot e + \sum_{i=1}^{2} \begin{bmatrix} e(t) \\ \dot e(t) \end{bmatrix}^T \begin{bmatrix} Q_i & U_i \\ U_i^T & R_i \end{bmatrix} \begin{bmatrix} e(t) \\ \dot e(t) \end{bmatrix} \\ &- \sum_{i=1}^{2}(1-\dot\tau_i(t)) \begin{bmatrix} e(t-\tau_i(t)) \\ \dot e(t-\tau_i(t)) \end{bmatrix}^T \begin{bmatrix} Q_i & U_i \\ U_i^T & R_i \end{bmatrix} \\ &\begin{bmatrix} e(t-\tau_i(t)) \\ \dot e(t-\tau_i(t)) \end{bmatrix} + \sum_{i=1}^{2} h_i \begin{bmatrix} e(t) \\ \dot e(t) \end{bmatrix}^T \begin{bmatrix} S_i & W_i \\ W_i^T & Z_i \end{bmatrix} \begin{bmatrix} e(t) \\ \dot e(t) \end{bmatrix} \\ &- \sum_{i=1}^{2}\int_{t-h_i(t)}^{t} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix}^T \begin{bmatrix} S_i & W_i \\ W_i^T & Z_i \end{bmatrix} \begin{bmatrix} e(s) \\ \dot e(s) \end{bmatrix} ds \end{aligned}$$

Since

$2e(t) - \sum_{i=1}^{2} e(t-\tau_i(t)) - \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \dot e(s)ds = 0$

and

$\dot e(t) - F_0 e(t) - F_1 e(t-\tau_1(t)) - F_2 e(t-\tau_2(t)) - TEf(t) = 0$

There exist free matrices H_i, $i = 1,7$ and free scalars ϵ_i, $i = 1,6$, $\bar\epsilon_i$, $i = 1,2$ and a matrix P such that:

$$\begin{aligned} &2\left[e(t)^T H_1 + e(t-\tau_1(t))^T H_2 + e(t-\tau_2(t))^T H_3 + \dot e(s)^T H_4 \right. \\ &\left. + \dot e^T(t-\tau_1(t))H_5 + \dot e^T(t-\tau_2(t))H_6 + f^T H_7\right] \\ &\times \left[2e(t) - \sum_{i=1}^{2} e(t-\tau_i(t)) - \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \dot e(s)ds\right] = 0 \end{aligned} \tag{24}$$

and

$$\begin{aligned} &2\left[e(t)^T\epsilon_1 + e^T(t-\tau_1(t))\epsilon_2 + e^T(t-\tau_2(t))\epsilon_3 + \dot e^T(s)\epsilon_4 \right. \\ &\left. + \dot e^T(t-\tau_1(t))\epsilon_5 + \dot e^T(t-\tau_2(t))\epsilon_6 + \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} e^T(s)ds\bar\epsilon_i\right] P \\ &\times \left[F_0 e(t) + F_1 e(t-\tau_1(t)) + F_2 e(t-\tau_2(t)) + TEf(t) - \dot e(t)\right] = 0 \end{aligned} \tag{25}$$

Defining $\Gamma_1^T = \begin{bmatrix} e^T & e^T(t-\tau_1) & e^T(t-\tau_2) & \dot e^T(t) & \dot e^T(t-\tau_1) & \dot e^T(t-\tau_2) & f^T(t) \end{bmatrix}$ and $\xi^T(s) = \begin{bmatrix} e^T(s) & \dot e^T(s) \end{bmatrix}$, (24) is equivalent to:

$$2\Gamma_1^T(t) H\Delta_1\Gamma_1(t) - 2\sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_1^T(t) \begin{bmatrix} 0 \\ H^T \end{bmatrix}^T \xi(s)ds = 0$$

where

$\Delta_1 = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$H^T = \begin{bmatrix} H_1^T & H_2^T & H_3^T & H_4^T & H_5^T & H_6^T & H_7^T \end{bmatrix}$

and (25) is equivalent to:

$$2\Gamma_1^T(t)\Upsilon\Delta_2\Gamma_1(t) - 2\sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_1(t)\bar H_i \xi(s)ds = 0$$

where

$\Upsilon^T = P^T \begin{bmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 & \epsilon_5 & \epsilon_6 & 0 \end{bmatrix}$

$\Delta_2 = \begin{bmatrix} F_0 & F_1 & F_2 & -I & 0 & 0 & TE \end{bmatrix}$

$\bar H_i = \begin{bmatrix} -\bar\epsilon_i\Delta_2^T P & H \end{bmatrix}$.

Let now consider

$$\dot g(e,f,s) = \dot V(t) + (24) + (25) + r^T(t)r(t) - \gamma^2 f^T(t)f(t)$$

$$\begin{aligned} \dot g(e,f,s) \le\ & \Gamma_1^T\Phi\Gamma_1 - \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \xi^T(s)\bar Z_i\xi(s)ds \\ & -2\sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_1^T\bar H_i\xi(s)ds \\ \dot g(e,f,s) \le\ & \Gamma_1^T(t)\Phi\Gamma_1(t) - \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \xi^T(s)\bar Z_i\xi(s)ds \\ & -2\sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_1^T(t)\bar H_i\xi(s)ds \\ \dot g(e,f,s) \le\ & \Gamma_1^T(t)\Phi\Gamma_1(t) \\ & - \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_2^T(t,s) \begin{bmatrix} 0 & \bar H_i \\ \bar H_i^T & \bar Z_i \end{bmatrix} \Gamma_2(t,s)ds \end{aligned}$$

In addition to this, we have the following true inequalities for any semipositive-definite matrices $\bar X_1$ and $\bar X_2$ of the form

$$\begin{bmatrix} \bar X_{11} & \bar X_{12} & \bar X_{13} & \bar X_{14} & \bar X_{15} & \bar X_{16} & \bar X_{17} \\ * & \bar X_{22} & \bar X_{23} & \bar X_{24} & \bar X_{25} & \bar X_{26} & \bar X_{27} \\ * & * & \bar X_{33} & \bar X_{34} & \bar X_{35} & \bar X_{36} & \bar X_{37} \\ * & * & * & \bar X_{44} & \bar X_{45} & \bar X_{46} & \bar X_{47} \\ * & * & * & * & \bar X_{55} & \bar X_{56} & \bar X_{57} \\ * & * & * & * & * & \bar X_{66} & \bar X_{67} \\ * & * & * & * & * & * & \bar X_{77} \end{bmatrix}$$

$$0 \le h_1\Gamma_1^T(t)\bar X_1\Gamma_1(t) - \int_{t-\tau_1(t)}^{t} \Gamma_1^T(t)\bar X_1\Gamma_1(t)ds \tag{26}$$

$$0 \le h_2\Gamma_1^T(t)\bar X_2\Gamma_1(t) - \int_{t-\tau_2(t)}^{t} \Gamma_1^T(t)\bar X_2\Gamma_1(t)ds \tag{27}$$

Adding (26), (27) to (28), we obtain:

$$\begin{aligned} &\dot g(e,f,s) \le \Gamma_1^T(\Phi + h_1\bar X_1 + h_2\bar X_2)\Gamma_1 \\ &- \sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t} \Gamma_2^T(t,s) \begin{bmatrix} \bar X_i & \bar H_i \\ \bar H_i^T & \bar Z_i \end{bmatrix} \Gamma_2(t,s)ds \end{aligned} \tag{28}$$

If we select: $\bar{X}_i = \bar{H}_i \bar{Z}_i^{-1} \bar{H}_i^T$, $\bar{X}_i \geq 0$ since $\bar{Z}_i > 0$ and by the Schur lemma, we have that

$$\Phi_i = \begin{bmatrix} \bar{X}_i & \bar{H}_i \\ \bar{H}_i^T & \bar{Z}_i \end{bmatrix} \geq 0$$

So, $\dot{g} < 0$ if $\Phi + h_1 \bar{H}_1 \bar{Z}_1^{-1} \bar{H}_1^T + h_2 \bar{H}_2 \bar{Z}_2^{-1} \bar{H}_2^T < 0$ which can be written as as

$$\begin{bmatrix} \Phi & h_1 \bar{H}_1 & h_2 \bar{H}_2 \\ h_1 \bar{H}_1^T & -h_1 \bar{Z}_1 & 0 \\ h_2 \bar{H}_2^T & 0 & -h_2 \bar{Z}_2 \end{bmatrix} < 0$$

and where

$$\begin{aligned}
&\Phi_{11} = \textstyle\sum_{i=1}^2 (h_i S_i + Q_i) + \epsilon_1 Sym(PF_0) + 2Sym(H_1) + C^T C \\
&\Phi_{12} = \epsilon_1 PF_1 + \epsilon_2 (PF_0)^T - H_1 + 2H_2^T \\
&\Phi_{13} = \epsilon_3 (PF_0)^T + \epsilon_1 PF_2 + 2H_3^T - H_1 \\
&\Phi_{14} = P + \textstyle\sum_{i=1}^2 (U_i + h_i W_i) + 2H_4^T + \epsilon_4 (PF_0)^T - \epsilon_1 P \\
&\Phi_{15} = 2H_5^T + \epsilon_5 (PF_0)^T \\
&\Phi_{16} = 2H_6^T + \epsilon_6 (PF_0)^T \\
&\Phi_{17} = 2H_7^T + \epsilon_1 PTE \\
&\Phi_{22} = -(1-d_1) Q_1 - Sym(H_2) + \epsilon_2 Sym(PF_1) \\
&\Phi_{23} = -H_3^T - H_2 + \epsilon_2 PF_2 + \epsilon_3 (PF_1)^T \\
&\Phi_{24} = -H_4^T + \epsilon_4 (PF_1)^T - \epsilon_2 P \\
&\Phi_{25} = -(1-d_1) U_1 - H_5^T + \epsilon_5 (PF_1)^T \\
&\Phi_{26} = -H_6^T + \epsilon_6 (PF_1)^T \\
&\Phi_{27} = -H_7^T + \epsilon_2 PTE \\
&\Phi_{33} = -(1-d_2) Q_2 - Sym(H_3) + \epsilon_3 (PF_2)^T \\
&\Phi_{34} = -H_4^T + \epsilon_4 (PF_2)^T \\
&\Phi_{35} = -H_5^T + \epsilon_5 (PF_2)^T \\
&\Phi_{36} = -(1-d_2) U_2 - H_6^T + \epsilon_6 (PF_2)^T \\
&\Phi_{37} = -H_7^T + \epsilon_3 PTE \\
&\Phi_{44} = \textstyle\sum_{i=1}^2 (R_i + h_i Z_i) + \epsilon_4 P \\
&\Phi_{45} = \epsilon_5 P \\
&\Phi_{46} = \epsilon_6 P \\
&\Phi_{47} = \epsilon_4 PTE \\
&\Phi_{55} = -(1-d_1) R_1 \\
&\Phi_{56} = 0 \\
&\Phi_{57} = \epsilon_5 PTE \\
&\Phi_{66} = -(1-d_2) R_2 \\
&\Phi_{67} = \epsilon_6 PTE \\
&\Phi_{77} = -\gamma^2 I_f
\end{aligned}$$

Now, substituting $F_i = \chi_i - K\beta_i$, $TE = \chi_f - K\beta_f$ and definig $U = PK$, we obtain the LMI (23). ■

IV. DIAGNOSIS SCHEME

The previous observer is insensitive to faults. In order to detect and isolate faults, we consider the generalized observer scheme where we use a bank of n_f robust H_∞ observers such that each observer$_j$ defined by eq. (29) is insensitive to only one fault f_j.

$$\left\{ \begin{array}{l} \dot{z}_j(t) = \sum_{i=0}^2 F_{i_j} z_j(t - \tau_i(t)) + \sum_{i=0}^2 T_j B_i u(t - \tau_i(t)) \\ + \sum_{i=0}^2 G_{i_j} y(t - \tau_i(t)) \\ \hat{x_j}(t) = z_j(t) + N_j y(t) \\ r_j(t) = y(t) - C\hat{x_j}(t) \end{array} \right. \tag{29}$$

The dynamic error estimation derived for each observer j is:

$$\begin{aligned} \dot{e}_j(t) = {} & \textstyle\sum_{i=0}^2 F_{i_j} e_j(t - \tau_i(t)) + T_j E_j f_j(t) \\ & + T_j \textstyle\sum_{l \neq j} E_l f_l(t) \\ r_j(t) = {} & C e_j(t) \end{aligned} \tag{30}$$

such as $\|T_{f_j r_j}\|_\infty < \gamma$.

This consists in generating a set of n_f observers related to a residual

$$r_j = y - C\hat{x}_j, \quad j = 1, 2, ..., n_f$$

which is insensitive to one element of the fault vector f_j (represented by 0) and sensitive to the $n_f - 1$ other components $f_l, l \neq j$ (represented by 1). That is summarized in the following table

If	r_1	r_2	$\cdots$	r_m
$f_1 \neq 0$	0	1	$\cdots$	1
$f_2 \neq 0$	1	0		$\vdots$
$\vdots$	$\vdots$	$\vdots$		1
$f_{n_f} \neq 0$	1	$\cdots$	1	0

.

In the next section, we will see the application of such diagnosis to our irrigation canal.

V. APPLICATION TO OPEN-CHANNEL OBSERVER DESIGN

The Integrator Delay model described by (11) is a low frequency approximate model of the Saint-Venant equations. The observer synthesis is based on this model. However, the application is tested on data obtained with SIC software (Simulation of Irrigation Canals) which is a hydraulic model solving the complete Saint-Venant equations [2], considered to be a very good physical model of an irrigation canal.

A. System description

As it was presented in Section I. The system is a series of two pools interconnected by regulation gates. Each pool is about 3000 m long. The delays are identical and equal to $\tau_1 = \tau_2 = 647s$ and $a_1 = a_2 = 3.21.10^{-5}$. The linearization coefficients are equal to: $\bar{b}_0 = \bar{b}_1 = \bar{b}_2 = -29.36m^2/s$, $b_0 = b_1 = b_2 = 29.05m^2/s$, $k_0 = k_1 = k_2 = 18.11m^2/s$. The potential faults are either on u_1 or on u_2 So, we have two faults.

B. Synthesis specifications

According to section IV, to achieve FDI on our system, we have to use a bank of two observers: observer 1 and observer 2 generating respectively the residuals r_1 and r_2 such that: $\|T_{r_1 f_1}\|_\infty < 1$ and $\|T_{r_2 f_2}\|_\infty < 1$.

In the sequel, we show the results obtained from the application of the previous diagnosis scheme based on the developed observer on the irrigation canal.

C. Results

For $\gamma = 1$, $\epsilon_1 = 10$, $\epsilon_2 = -1$, $\epsilon_3 = -1$, $\epsilon_4 = -10$, $\epsilon_5 = -1$, $\epsilon_6 = -1$, $\bar{\epsilon}_1 = -1$ and $\bar{\epsilon}_2 = -1$, we have obtained the following parameters of the observer$_1$:

$$T_1 = \begin{bmatrix} -0.001 & 0.002 \\ -0.000 & 0.490 \end{bmatrix}, F_{01} = \begin{bmatrix} -0.638 & -0.000 \\ 0.000 & -0.6721 \end{bmatrix},$$

$$F_{11} = \begin{bmatrix} 0.043 & 0.000 \\ -0.000 & 0.076 \end{bmatrix}, F_{21} = \begin{bmatrix} -0.027 & 0.000 \\ 0.000 & -0.004 \end{bmatrix},$$

$$G_{01} = \begin{bmatrix} 0.0006 - 0.0002 \\ -0.00000.0340 \end{bmatrix}, G_{11} = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.036 \end{bmatrix},$$

$$G_{21} = \begin{bmatrix} 0.001 & -0.000 \\ 0.297 & 0.000 \end{bmatrix}, N_1 = \begin{bmatrix} 0.982 & -0.002 \\ 0.000 & 0.509 \end{bmatrix},$$

We can see in fig.2 (respectively in fig.3), the transfer function between the residual r_1 (respectively r_2) and the faults before (in continous line) and after (in dashed line) applying the observer$_1$ (respectively, the observer$_2$).

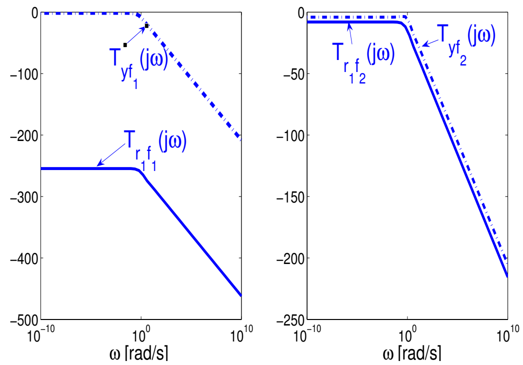

Fig. 2. Transfer between residue r_1 and faults f_1 and f_2

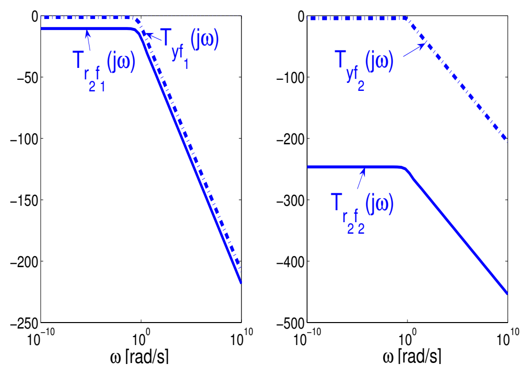

Fig. 3. Transfer between residue r_2 and faults f_1 and f_2

In order to simulate faults, we have generated a biased fault of 3 cm non simultaneously on each opening gate u_1 and u_2 such that the fault f_1 occurs after 3 hours then the second fault f_2 occurs after 9 hours. In fig. 4, we show the resulting residual denoted r_1 and r_2 corresponding to the outputs of observer$_1$ and observer$_2$. We can observe that

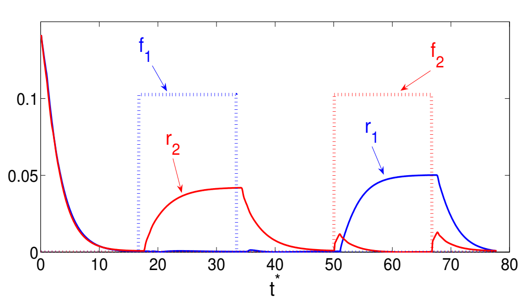

Fig. 4. Test on the SIC model

before 3 h which indicates the absence of faults, the residuals tend to go to zero which corresponds to a correct state estimation. Then, at 3 h, the fault f_1 occurs on the opening gate u_1. The observer$_1$ designed to be insensitive to f_1 keeps the residual r_1 to zero where the residual r_2 generated by the observer$_2$ which is sensitive to f_2 is different from zero. The same phenomena happens when the fault f_2 occurs leading to r_1 different from zero and r_2 null. It is well shown that the diagnosis scheme is achieved successfully according to the signature table.

If	r_1	r_2
$f_1 \neq 0$	0	1
$f_2 \neq 0$	1	0

VI. Conclusion

In this paper, we have presented a method for the design of a H_∞ observer for time-varying delayed state and inputs system. Existence conditions, delay-dependent stability conditions of the observer have been given and proved. Then, a fault detection and isolation scheme (FDI) based on a bank of such observer has been proposed and tested on an open-channel with two pools in series in order to detect and isolate non simultaneous faults. The observer design is based on a low frequency approximate model of Saint-Venant equations, which has been modified to become dimensionless in order to consider a generic problem. The results obtained showed that the diagnosis objective has been well achieved.

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